2. Let e=eccentricity. Then area$=\pi a^2\sqrt{(1-e^2)}$.

$$\therefore \text{ Average area}=\pi a^2\frac{\int_0^1\sqrt{(1-e^2)}}{\int_0^1 de}=\tfrac{1}{4}\pi^2a^2=2.4674a^2.$$

MISCELLANEOUS.

73. Proposed by CHAS. E. MYERS, Canton, Ohio.

In an ice cream freezer, cream of a homogeneous character and at the uniform temperature of 60° Fahrenheit is put into a cylinder having a closed base, and the whole put into a freezing mixture so as to subject the base and convex surface to a constant temperature of 30° Fahrenheit. Required the temperature at any point within the cream after the expiration of a given time. [From *Higher Mathematics*.]

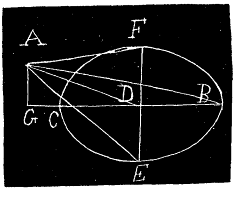
No solution of this problem has been received.

74. Proposed by S. HART WRIGHT, M. D., A. M., Ph. D., Penn Yan, N. Y.

The longest diameter of a horizontal ellipse is $CB=2a=6$ feet. Its shortest diameter is $EF=2b=4$ feet, their intersection being at D. Find in an indefinite vertical plane passing through CB, a point $A=5$ feet$=c$ from D, the ellipse being seen from A as a circle.

I. Solution by the late B. F. BURLESON, and the PROPOSER.

The eye being at A, and the ellipse being projected as a circle, CB and EF subtend equal angles at A, or $\angle EAF=\angle BAC$. Produce DC to G, A being vertically over G, and put $CG=x$, and $GA=y$, and $\angle ADC=\phi$=angle of elevation of A.



Then $y=\sqrt{[c^2-(a+x)^2]}\ldots\ldots(1)$.

$AB=\sqrt{[(2a+x)^2+y^2]}\ \ldots\ldots(\alpha)$.

$\sin\angle ACG=\sin\angle ACB=y/\sqrt{(x^2+y^2)}\ldots\ldots(\beta)$. and $\tan\angle EAD=b/c\ \ldots\ldots(\gamma)$.

$\therefore\ \sin\angle EAF=\sin\angle BAC=2dc/(b^2+c^2)\ldots\ldots(\delta)$.

From $\triangle BAC$ we have the proportion, $AB:\sin\angle ACB::BC:\sin\angle BAC$.

$$\therefore\ \frac{2bc\sqrt{[(2a+x)^2+y^2]}}{b^2+c^2}=\frac{2ay}{\sqrt{(x^2+y^2)}}\ldots\ldots(2).$$

Resolving (1) and (2) we have $x=\dfrac{c\sqrt{[(c^4-a^2b^2)(a^2-b^2)]}-a}{a(c^2-b^2)}$

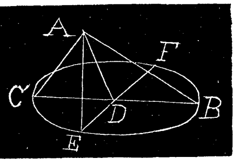
$$=\tfrac{5}{63}\sqrt{(2945)}-3=1.30697255 \text{ feet.}$$

$$\therefore\ y=\frac{bc(c^2-a^2)}{a(c^2-b^2)}=2\tfrac{34}{63}\text{ feet.}$$

$$\phi=\sin^{-1}\left(\frac{b(c^2-a^2)}{a(c^2-b^2)}\right)=\sin^{-1}(\tfrac{32}{63})=30^\circ\ 31'\ 35\tfrac{1}{4}''.$$

II. Solution by J. W. YOUNG, Fellow and Assistant in Mathematics, Ohio State University, Columbus, Ohio.

If the ellipse is seen from A as a circle, $\angle EAF=\angle CAB$. This relation enables us to calculate the coördinates of the point A, in the vertical plane through CB, the origin being at D, and CB being the axis of x.



$$\tan\tfrac{1}{2}\angle EAF=b/c.$$

Also let $AC=q$ and $AB=p$; $CB=2a$.

Then $\tan\frac{1}{2}\angle CAB=\sqrt{\left(\frac{(s-p)(s-q)}{s(s-2a)}\right)}$

where $s=\frac{1}{2}(p+q+2a)$.

Since $\angle EAF=\angle CAB$, we have, after reducing,

$$\frac{b^2}{c^2}=\frac{4a^2-(p-q)^2}{(p+q)^2-4a^2}\ldots\ldots(1).$$

Now, since $AD=c$ is the median of $\triangle ABC$, we have by a common trigonometrical formula,

$$2c^2=p^2+q^2-2a^2 \text{ or } p^2+q^2=2(a^2+c^2)\ldots\ldots(2).$$

From equations (1) and (2), we obtain

$$p+q=\pm2\sqrt{\left(\frac{a^2b^2-c^4}{b^2-c^2}\right)};\ p-q=\pm2c\sqrt{\left(\frac{b^2-a^2}{b^2-c^2}\right)}.$$

Now, to obtain x and y, we have the relations,

$$(a+x)^2+y^2=p^2,\ (a-x^2)+y^2=q^2,$$

from which $x=\dfrac{p^2-q^2}{4a}$.

Substituting from above, we have

$$x=\frac{\pm c}{a(b^2-c^2)}\sqrt{[(a^2b^2-c^4)(b^2-a^2)]},\quad y=\pm\frac{bc(a^2-c^2)}{a(b^2-c^2)}.$$

In the special case given, where $a=3$, $b=2$, $c=5$, we have

$$x=\pm4.31 \text{ feet},\quad y=\pm2.54 \text{ feet}.$$

NOTE.—Other solutions of this problem will appear next month.